STATISTICS I - 2nd Year Economics\Management Science BSc – 2nd semester – **30/05/2016**

1st Part of Normal Season Exam– Theoretical Part (15 minutes)

This exam consists of two parts. This is Part 1 - Theoretical (35 points). During the exam, no clarifications will be provided. **GOOD LUCK!**

Name: _____ **nº**_____

Each of the following 2 groups of multiple-choice questions is worth 10 points (1 mark). Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade in each of the 2 groups varies between a minimum of 0 and a maximum of 10 points.

Indicate whether the following statements are true **(T)** or false **(F)** by ticking the corresponding box with a cross **(X)**

1. Let, $A, B \subset S$ be events of a sample space S with positive probability. It is known that when A occurs, B does not occur.

	T	F
A and B are a partition of the sample space S		X
Then A and B are independent events.		X
$P[\bar{A} \cap \bar{B}] = 1 - P(A).P(B)$		X
$P(A - B) \leq P(A)$	X	

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	T	F
If X is discrete, $\forall\, h > 0, x \in \mathbb{R}$, then $F_X(x) \leq P(X \leq x + h)$	X	
Let $Y = \varphi(X)$ be a function of X. If X is a discrete random variable, then Y can be a mixed random variable.		X
Let X be discrete, then $F_X(x)$ has range $\mathfrak{R}$ and co-domain [0,1].	X	
If $a, b \in D_X, a < b$ then $P(a \leq X \leq b) = F_X(b) - F_X(a - 0)$	X	

3. Let A, B be events of a sample space S. Assuming that A and B are mutually exclusive events show that $P(B|A \cup B) = 1 - P(A)/P(A \cup B)$. **Note: this question should be duly formalized and justified**. [Cotação: 15]

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Indicate whether the following statements are true **(T)** or false **(F)** by ticking the corresponding box with a cross **(X)**

1. Let, $A, B \subset S$ be events of a sample space S with positive probability. It is known that events A and B can occur simultaneously..

	T	F
$P(A-B) \leq P(A)$	X	
$P[\bar{A} \cap \bar{B}] = 1 - [P(A) + P(B)]$		X
If $P(A\|B) = P(A)$ then A and B are independent events.	X	
A and B are a partition of the sample space S		X

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	T	F
If X is continuous, $\forall\, h > 0, x \in \mathbb{R}$ then $F_X(x) \leq P(X \leq x + h)$	X	
Let $Y = \varphi(X)$ be a function of X. If X is a mixed random variable, then Y can be a continuous random variable.		X
Let X be continuous, then $f_X(x)$ has range $\Re$ and co-domain $[0, +\infty]$.	X	
If $a, b \in D_X, a < b$ then $P(a < X < b) = F_X(b) - F_X(a-0)$		X

3. Let A, B be events of a sample space Ω. Assuming that A and B are mutually exclusive events show that $P(B|A \cup B) = 1 - P(A)/P(A \cup B)$. **Note**: **this question should be duly formalized and justified**. [Cotação: 15]

STATISTICS I - 2nd Year Economics\Management Science BSc – 2nd semester – **30/05/2016**
1st Part of Normal Season Exam – Practical Part (45 minutes)

This is Part 2: 12 marks. The answers to the multiple-choice questions should be given by signalling with an **X** the corresponding square. The other questions should be answered in the provided space.

Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.
Open questions should be duly justified and formalized.

Name:_____Nº:_____

Espaço reservado para a classificação			
1 a) (10) b) (15) _____	2 a) (10) b) (15) _____	2 c) (15) _____	T: P: _____

1. The two more important wine producers (V_1 and V_2) produce respectively 30% and 40% of all the wine bottles bought by a restaurant. The owner of the restaurant noticed that 20% of the bottles bought from V_1 and 15% of those bought from V_2 has a minor quality. It is also known that the percentage of wine of minor quality from other producers is 10%.

a) If 10 bottles were randomly chosen from the restaurant stock, with replacement, compute the probability that 6 of them came from wine producer V_2. (signal with an X the right answer,)

(i) 0,5956 ☐ (ii) 0,1797 ☐ (iii) 0,9452 ☐ (iv) 0,1115 X

b) A bottle was randomly chosen and it was of minor quality. Find the probability that it came from producer V_1.

$$\boldsymbol{P(V1) = 0.3;\ P(V2) = 0.4;\ P(M|V1) = 0.2;\ P(M|V2) = 0.15;}$$

2. Let (X, Y) be a two dimensional continuous random variable with joint probability density function given by:

$$f_{X,Y}(x,y) = \begin{cases} kx + y & (0 < x < 1, \quad 0 < y < 1) \\ 0 & elsewhere \end{cases}$$

a) Find the value of k.

b) Determine the marginal cumulative distribution function of X and **use it** to compute the 1st Quartile.

c) Compute the $E\left(X|Y = \frac{1}{2}\right)$.

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1st Part of Normal Season Exam – Practical Part (45 minutes)

This is Part 2: 12 marks. The answers to the multiple-choice questions should be given by signalling with an **X** the corresponding square. The other questions should be answered in the provided space.

Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.
Open questions should be duly justified and formalized.

Name:_____ Nº:_____

Espaço reservado para a classificação				
	a) (10)	a) (10)	2 c) (10)	T:
1		2		
	b) (15)	b) (20)		P:
	_____	_____	_____	_____

1. The two more important wine producers (V_1 and V_2) produce respectively 40% and 30% of all the wine bottles bought by a restaurant. The owner of the restaurant noticed that 20% of the bottles bought from V_1 and 15% of those bought from V_2 has a minor quality. It is also known that the percentage of wine of minor quality from other producers is 10%.

a) If 20 bottles were randomly chosen from the restaurant stock, with replacement, compute the probability that 8 of them came from wine producer V_1. (signal with an X the right answer,)

(i) 0,5956 ☐ (ii) 0,1797 X (iii) 0,9452 ☐ (iv) 0,1115 ☐

b) A bottle was randomly chosen and it was of minor quality. Find the probability that it came from producer V_2.

2. Let (X, Y) be a two dimensional continuous random variable with joint probability density function given by:

$$f_{X,Y}(x,y) = \begin{cases} kx + y & (0 < x < 1, \quad 0 < y < 1) \\ 0 & elsewhere \end{cases}$$

a) Find the value of k.

b) Determine the marginal cumulative distribution function of Y and **use it** to compute the 1st Quartile.

d) Compute the $E\left(Y|X = \frac{1}{2}\right)$.